# QNO : 1 & 2 IN IPE

# 1. CIRCLES

1. Find the centre and radius of the circle $x^2+y^2-4x-8y-41=0$ **[March 2020TS]**
2. Find the equation of the circle with centre C(1,4) and radius r = 5 **[March 2017AP]**
3. Find the equation of the circle passing through
   i). the origin has centre at (-4,-3) **[March 2004]**
   ii). (2,-1) and having centre at (2,3) **[May 2008]**
   iii). (3,4) and having centre at (-3,4) **[May 2018AP March 2012]**
   iv). (-2,3) and having centre at (0,0) **[May 2015TS]**
   v). (5,6) and having centre at (-1,2) **[March 2018TS]**
4. i). Find the value of 'a' if $2x^2+ay^2-3x+2y-1=0$ represents a circle and find the radius of the circle. **[March 2015AP 2013]**
   ii). If $x^2+y^2+2gx+2fy-12=0$ is a circle with centre (2,3) then find radius of the circle. **[May 2011]**
   iii). If $x^2+y^2+2gx+2fy=0$ is a circle with centre (-4,-3) then find radius of the circle. **[May 2017AP]**
   iv). If $x^2+y^2+ax+by-12=0$ is a circle with centre (2,3) find a, b and the radius of this circle **[March 2008 May 2009]**
   v). If $x^2+y^2-4x+6y+a=0$ is a circle of radius 4, find the value of 'a' **[March 2016AP]**
   vi). If $x^2+y^2-4x+6y+c=0$ is a circle of radius 6, find the value of 'c' **[May 2018TS]**
5. Find the power of the point (-1,1) with respect to $x^2+y^2-6x+4y-12=0$ **[March 2016TS]**
6. Find the equation of the circle having $\overline{AB}$ as diameter where A, B are
   i). (-4,3), (3,-4) **[May 2019TS 2013]**
   ii). (4,2), (1,5) **[March 2019AP]**
7. Find the other end of the diameter of the circle $x^2+y^2-8x-8y+27=0$ if one end of it is (2,3) **[May 2012 March 2020AP]**
8. Find the equation of the circle concentric with $x^2+y^2-6x-4y-12=0$ and passing through (-2,14) **[May 2017TS 2014 March 2014]**
9. Find the area of the triangle formed with the coordinate axes and the tangent drawn at the point $(x_1,y_1)$ on the circle $x^2+y^2=a^2$ **[March 2020AP]**
10. Find the equation of the normal to the circle $x^2+y^2-10x-2y+6=0$ at (3,5) **[March 2018AP]**
11. Find the length of the tangent from the point (-2,5) to the circle $x^2+y^2=25$ **[May 2016AP]**
12. Find the value of k, if the length of the tangent from the point
    i). (5,4) to the circle $x^2+y^2+2ky=0$ is equal to 1 **[May 18AP 16AP 15AP 15TS March 20TS 15AP 15TS]**
    ii). (2,5) to the circle $x^2+y^2-5x+4y+k=0$ is equal to $\sqrt{37}$ **[May 18TS 17AP March 18TS]**
13. Find the angle between the tangents from the point (4,-2) to the circle $x^2+y^2=10$ **[July 2001]**

14. Define chord of contact and find the chord of contact of (1,1) to the circle $x^2+y^2 = 9$ **[March 2020AP]**

15. Find the polar of the point
    i). (1,-2) with respect to $x^2+y^2-10x-10y+25 = 0$ **[March 2015TS]**
    ii). (3,-1) with respect to $2x^2+2y^2 = 11$ **[May 2019TS]**

16. Find the pole of the line $lx+my+n = 0$ with respect to $x^2+y^2 = a^2$ **[May 2016AP]**

17. Show that the points
    i). (4,-2),(3,-6) are conjugate with respect to the circle $x^2+y^2 = 24$ **[March 2004]**
    ii). (4,2),(3,-5) are conjugate with respect to the circle $x^2+y^2-3x-5y+1= 0$ **[April 96]**

18. Find the value of k if the points
    i).(1,3),(2,k) are conjugate with respect to $x^2+y^2 = 35$
    **[May 2017TS March 2019TS 2017AP 2016TS]**
    ii).(2,3),(4,k) are conjugate with respect to $x^2+y^2 = 17$ **[Oct 95]**
    iii).(4,2),(k,-3) are conjugate with respect to $x^2+y^2-5x+8y+6 = 0$
    **[March 2019AP 2017TS May 2014]**

19. Write the parametric equations of the circle
    i).$x^2+y^2 = 4$ **[March 2014]**
    ii).$4(x^2+y^2) = 9$ **[March 2017TS]**
    iii).$x^2+y^2-6x+4y-12 = 0$ **[May 2015AP]**
    iv).$(x-3)^2+(y-4)^2 = 8^2$ **[March 2018AP 2016AP]**
    v).$2x^2+2y^2 = 7$ **[March 2019TS]**

## QNO : 3 IN IPE

## 2. SYSTEM OF CIRCLES

1. Show that the circles $x^2+y^2-2x-2y-7 = 0$, $3x^2+3y^2-8x+29y = 0$ cut each other orthogonally. **[May 2015AP]**

2. Find 'k' if the following circles are orthogonal
    i). $x^2+y^2+4x+8 = 0$, $x^2+y^2-16y+k = 0$ **[March 2016AP 2016TS]**
    ii). $x^2+y^2-5x-14y-34 = 0$, $x^2+y^2+2x+4y+k = 0$ **[May 2018TS March 2020 AP 2018AP]**

3. Find the equation of the circle which cuts orthogonally the circle $x^2+y^2-4x+2y-7 = 0$ and having the centre at (2, 3) **[March 2019TS]**

4. Find the angle between the circles
    i). $x^2+y^2+4x-14y+28 = 0$, $x^2+y^2+4x-5 = 0$ **[May 2017AP]**
    ii). $x^2+y^2-12x-6y+41 = 0$, $x^2+y^2+4x+6y-59 = 0$ **[May 2018AP 2015TS 2017TS 2015TS]**
    iii). $x^2+y^2 = a^2$, $x^2+y^2 = ax+ay$ **[May 2016TS March 2020TS 2014]**

5. Find the equation of the radical axis of the two circles
    i). $x^2+y^2-2x-4y-1 = 0$, $x^2+y^2-4x-6y+5 = 0$ **[May 2016AP]**
    ii). $3x^2+3y^2-7x+8y-11 = 0$, $x^2+y^2-3x-4y+5 = 0$ **[May 2017TS]**
    iii). $2x^2+2y^2+3x+6y-5 = 0$, $3x^2+3y^2-7x+8y-11 = 0$ **[March 2017AP]**

iv). $x^2+y^2+4x+6y-7=0$, $4x^2+4y^2+8x+12y-9=0$ **[March 2019AP 2019TS]**

6. Find the equation of the common chord of the two circles

i).$(x-a)^2+(y-b)^2=c^2$, $(x-b)^2+(y-a)^2=c^2$ **[March 2015AP]**

ii).$x^2+y^2-4x-4y+3=0$, $x^2+y^2-5x-6y+4=0$ **[March 2020TS]**

7. Find the radical centre of the circles

i). $x^2+y^2-4x-6y+5=0$, $x^2+y^2-2x-4y-1=0$, $x^2+y^2-6x-2y=0$ **[May 2018TS March 2018AP]**

ii). $x^2+y^2+4x-7=0$, $2x^2+2y^2+3x+5y-9=0$, $x^2+y^2+y=0$ **[May 2016TS 2014]**

# QNO : 4 IN IPE

## 3. PARABOLA

1. Find the coordinates of the point on the parabola

i). $y^2=8x$ whose focal distance is 10 **[May 17TS March 17AP 17TS 16AP 2014]**

ii). $y^2=2x$ whose focal distance is 5/2 **[May 2015AP March 2015AP 2013]**

2. Find the equation of the parabola whose

i). focus is (1,-7), vertex is (1,-2) **[March 2015TS May 2015TS 2012]**

ii). focus is (3,1), vertex is (3,-2) **[May 2017AP March 2020AP 2018TS]**

3. i).Find the equation of the tangent to the parabola $x^2-4x-8y+12=0$ at (4,3/2) **[March 19AP]**

ii).Find the equation of the tangent to the parabola $y^2=6x$ at the positive end of the latus rectum. **[March 2020TS]**

4. Find the value of k if the line $2y=5x+k$ touches the parabola $y^2=6x$ **[May 18TS March 18AP 16TS]**

ii). Find the equation of the tangent to the parabola $y^2=16x$ inclined at $60^0$ to the x-axis. Find its point of contact. **[May 2016AP]**

5. If the chord joining the points $t_1$ and $t_2$ on the parabola $y^2=4ax$ is a focal chord then $t_1t_2=-1$. **[March 2003]**

6. If the normal at '$t_1$' on the parabola $y^2=4ax$, meets it again at $t_2$ then show that $t_2=-t_1-2/t_1$ **[May 2013]**

7. If a normal chord at point 't' on the parabola $y^2=4ax$, subtends a right angle at the vertex, then show that $t=\pm\sqrt{2}$ **[May 2014]**

8. If (1/2, 2) is one extremity of a focal chord of the parabola $y^2=8x$, then find the other extremity. **[May 2018AP 2016TS 2014]**

9. Find the equation of the normal to the parabola $y^2=4x$ which is parallel to $y-2x+5=0$ **[March 2019TS]**

# QNO : 5 IN IPE

## 5. HYPERBOLA

1. Define rectangular hyperbola and find its eccentricity. **[March 2015AP 2014]**

2. i). If e, $e_1$ are the eccentricities of a hyperbola and its conjugate, prove that $\frac{1}{e^2}+\frac{1}{e_1^2}=1$ **[May 2018TS March 2017TS 2011]**

ii). If the eccentricity of a hyperbola is 5/4 then find the eccentricity of its conjugate hyperbola. **[May 16AP 15AP 15TS 13 March 19TS 17AP 16AP 15TS 13 12]**

3. Find the equation of the hyperbola whose foci are $(\pm 5,0)$ the transverse axis is of length 8. **[March 2016TS May 2018AP 2011]**

4. If $3x-4y+k=0$ is a tangent to $x^2-4y^2=5$ find the value of k. **[May 2017TS March 2020AP 2018TS 2017TS]**

5. If the angle between the asymptotes of the hyperbola is $30^0$ then find its eccentricity. **[May 2017AP 2014 March 2020TS]**

6. Find the product of lengths of the perpendiculars from any point on the hyperbola $x^2/16 - y^2/9 = 1$ to its assymptotes. **[May 2019TS 2016TS March 2019AP]**

# QNO : 6 & 7 IN IPE

# 6. INDEFINITE INTEGRATION

Evaluate

1. i). $\int \frac{(3x+1)^2}{2x}\,dx$ **[May 2018AP 2016AP]** ii).$(x+1/x)^3\,dx$ **[March 2012]**

2 i). $\int \sec^2 x \csc^2 x\,dx$ **[May 18TS 17AP March 16TS]**

ii). $\int\left(x+\frac{4}{1+x^2}\right)dx$ **[May 2015TS]** iii). $\int\left(\frac{1}{\sqrt{1-x^2}}+\frac{2}{\sqrt{x^2+1}}\right)dx$ **[May 2011]**

iv). $\int \frac{1}{\cosh x+\sinh x}\,dx$ **[March 2017TS 2016AP]** v). $\int \frac{\cos x+\sin x}{\sqrt{1+\sin 2x}}\,dx$ **[April 98]**

vi). $\int \frac{1+\cos^2 x}{1-\cos 2x}\,dx$ **[March 2019TS 2013]** vii). $\int \frac{\sin^2 x}{1+\cos 2x}\,dx$ **[March 20TS]**

viii). $\int \frac{1}{1+\cos x}\,dx$ **[March 2015TS]** ix). $\int \sqrt{1-\cos 2x}\,dx$ **[March 2009]**

x). $\int \sqrt{1+\cos 2x}\,dx$ **[March 94]** xi). $\int \sqrt{1-\sin 2x}\,dx$ **[May 2017TS]**

3. i). $\int x^3 \sin x^4\,dx$ **[April 2001]** ii). $\int e^x \sin e^x\,dx$ **[March 17AP]**

iii). $\int \frac{2x+1}{x^2+x+1}\,dx$ **[March 2020TS]** iv). $\int \frac{1}{x\log x}\,dx$ **[Oct 99]**

v). $\int \frac{dx}{x\log x[\log(\log x)]}$ **[March 2019TS]** vi). $\int \frac{e^x}{e^x+1}\,dx$ **[March 18TS]**

4. i). $\int \frac{1}{\sqrt{x}}\cos\sqrt{x}\,dx$ **[April 98]** ii). $\int\left(1-\frac{1}{x^2}\right)e^{x+(1/x)}\,dx$ **[May 2012]**

iii). $\int \frac{(1+x)e^x}{\cos^2(xe^x)}\,dx$ **[May 2017AP 2016TS March 2019AP 2017TS 2010]**

5. i). $\int \frac{\cot(\log x)}{x}\,dx$ **[March 2005]** ii). $\int \frac{\log(1+x)}{1+x}\,dx$ **[March 2015TS]**

6. i). $\int \cos^3 x \sin x \, dx$ **[March 2018TS]** ii). $\int \frac{\sin^4 x}{\cos^6 x} dx$ **[March 2011]**

7. i). $\int \frac{dx}{\sqrt{\sin^{-1} x}\sqrt{1-x^2}}$ **[May 2015AP]** ii). $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$ **[March 18AP 15AP]**

8. i). $\int \frac{x^8}{1+x^{18}} dx$ **[May 19TS March 16AP]** ii). $\int \frac{2x^3}{1+x^8} dx$ **[May 2008]**

9. $\int \frac{\cos x}{(1+\sin x)^2} dx$ **[March 2020AP]**

10. i). $\int \left(\frac{1}{1-x^2} + \frac{1}{1+x^2}\right) dx$ **[May 2016TS]** ii). $\int \frac{1}{(x+1)(x+2)} dx$ **[March 19AP]**

11. $\int \frac{1}{\sqrt{x^2+2x+10}} dx$ **[May 2006]**

12. i). $\int \frac{1}{(x+5)\sqrt{x+4}} dx$ **[March 2002]** ii). $\int \frac{1}{(x+3)\sqrt{x+2}} dx$ **[March 2014]**

iii). $\int \frac{1}{(x+2)\sqrt{x+1}} dx$ **[April 99]**

13. $\int \frac{\cos x}{\sin^2 x + 4\sin x + 5} dx$ **[March 2007]**

14. i). $\int \log x \, dx$ **[June 2010]** ii). $\int x \log x \, dx$ **[March 20AP]**

iii). $\int \frac{1}{x^2} \log x \, dx$ **[May 2018AP]** iv). $\int \sqrt{x} \log x \, dx$ **[March 16TS]**

15. i). $\int e^x (\sin x + \cos x) dx$ **[March 2017AP]** ii). $\int e^x (\tan x + \sec^2 x) dx$ **[March 2006]**

iii). $\int e^x \sec x (1+\tan x) dx$ **[May 2016AP]** iv). $\int e^x (\tan x + \log \sec x) dx$ **[May 18TS 15TS]**

v). $\int e^x \left(\frac{1+x\log x}{x}\right) dx$ **[May 2019TS March 2018AP 2015AP 2013]**

vi). $\int e^x \frac{(x+1)}{(x+2)^2} dx$ **[May 2009]** vii). $\int \frac{xe^x}{(x+1)^2} dx$ **[May 2014]**

17. i). $\int e^x \cos x \, dx$ **[May 2015AP]** ii). $\int e^{ax} \sin(bx+c) dx$ **[March 19TS]**

## QNO : 8 & 9 IN IPE

## 7. DEFINITE INTEGRALS

Evaluate

1. i). $\int_0^a (\sqrt{a} - \sqrt{x})^2 dx$ **[March 2019TS]** ii). $\int_0^4 \frac{x^2}{1+x} dx$ **[May 2015TS]**

2. i). $\int_0^{\pi} \sqrt{2+2\cos\theta}\ d\theta$ **[March 18AP 16AP May 16AP]** ii). $\int_0^{\pi/4} \sec^4 x\ dx$ **[May 2014]**

3. i). $\int_2^3 \frac{2x}{1+x^2} dx$ **[March 20TS 17AP 16TS]** ii). $\int_0^a \frac{1}{x^2+a^2} dx$ **[May 19TS 15AP]**

iii). $\int_0^1 \frac{x^2}{1+x^2}\ dx$ **[May 2018TS]** iv). $\int_0^3 \frac{x}{\sqrt{x^2+16}}\ dx$ **[March 2017TS]**

v). $\int_0^2 \sqrt{4-x^2}\ dx$ **[March 2007]** vi). $\int_0^a \sqrt{a^2-x^2}\ dx$ **[March 2016TS]**

vii). $\int_0^1 \frac{dx}{\sqrt{3-2x}}\ dx$ **[March 2019AP]** viii). $\int_1^5 \frac{dx}{\sqrt{2x-1}}\ dx$ **[March 2015TS]**

4. $\int_0^{\pi/2} x \sin x\, dx$ **[March 2018TS]**

5. i). $\int_0^2 |1-x|\ dx$ **[May 2017AP 2016TS 2011 March 2018TS 2015AP]**

ii). $\int_0^4 |2-x|\ dx$ **[May 2017AP 2013]** iii). $\int_{-\pi/2}^{\pi/2} \sin|x|\ dx$ **[March 2017TS]**

6. i). $\int_0^{\pi/2} \sin^4 x\, dx$ **[May 2006]** ii). $\int_0^{\pi/2} \sin^7 x\, dx$ **[March 2017AP]**

iii). $\int_0^{\pi/2} \cos^{11} x\, dx$ **[March 2019TS]**

7. i). $\int_0^{\pi/2} \sin^4 x \cos^5 x\, dx$ **[March 2010]** ii). $\int_0^{\pi/2} \sin^5 x \cos^4 x\, dx$ **[March 2015AP]**

iii). $\int_0^{\pi/2} \sin^6 x \cos^4 x\, dx$ **[May 2016AP March 2019AP]**

8. i). $\int_0^{\pi} \sin^3 x \cos^3 x\, dx$ **[May 2015TS]** ii). $\int_{-\pi/2}^{\pi/2} \sin^3 x \cos^3 x\, dx$ **[May 2014]**

iii). $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^4 x\, dx$ **[March 20AP 18AP 16AP May 16TS]**

iv). $\int_0^{2\pi} \sin^4 x \cos^6 x \, dx$ **[May 17AP March 19TS]**

v). $\int_0^{2\pi} \sin^2 x \cos^4 x \, dx$ **[May 18AP March 15TS 14]**

9. i). Find the area of the region bounded by the parabola $y = x^2$, x-axis and the lines $x = -1$, $x = 2$. **[May 2018TS 2016TS 2015AP]**

ii). Find the area enclosed between the curve $y = x^3+3$, $y = 0$ and $x = -1$, $x = 2$ **[May 2017TS March 2020TS 2012]**

10. Find the area cut off between $x = 0$, and $x = 4 - y^2$ **[March 2011 2010]**

11. Find the area cut off between $x = 0$, and $2x = y^2 - 1$ **[May 2012]**

## QNO : 10 IN IPE

## 8. DIFFERENTIAL EQUATIONS

1. Find the order and degree of the differential equation

i). $\left(\frac{d^3y}{dx^3}\right)^2 - 3\left(\frac{dy}{dx}\right)^2 - e^x = 4$ **[March 2014]**

ii). $x^{1/2}\left(\frac{d^2y}{dx^2}\right)^{1/3} + x\frac{dy}{dx} + y = 0$ **[March 2018AP 2015TS]**

iii). $\left(\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3\right)^{6/5} = 6y$ **[May 2017TS 2015AP 2015TS 2011 March 2016AP 2013]**

iv). $\frac{d^2y}{dx^2} = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{5/3}$ **[May 2018TS]**

2. i). From the differential equation corresponding to $y = cx - 2c^2$ where c is parameter. **[March 2019AP 2012]**

ii). From the differential equation satisfying $y = A\cos 3x + b \sin 3x$ where A and B are parameters. **[March 2020TS 2015AP May 2016TS 2014]**

iii).From the differential equation from $y = a\cos(nx+b)$ by eliminating the constants a, b. **[May 2017AP]**

iv).From the differential equation from the relation $y = c(x - c)^2$ by eliminating the arbitary constant 'c' **[March 2016TS]**

3. From the differential equation of the family of all circles with their centres at the origin and also find its order. **[March 2017TS 2011]**

4. Solve the following differential equations

i). $x + y\frac{dy}{dx} = 0$ **[May 2016AP]**

ii). $\frac{dy}{dx}=\frac{2y}{x}$ **[March 2019TS 2017AP]**

iii). $\sqrt{1+x^2}\ dx + \sqrt{1+y^2}\ dy = 0$ **[March 2009]**

iv). $\frac{dy}{dx} = e^{x-y}+x^2\ e^{-y}$ **[May 18AP 17TS]**

v). $\frac{dy}{dx} = e^{x+y}$ **[March 2018TS]**

vi). Solve $y(1+x)\ dx + x(1+y)\ dy = 0$ **[March 2020AP]**

vii). $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ **[May 2019TS]**

# QNO : 11 IN IPE
# 1. CIRCLES

1. If the abscissae of the two points A and B are the roots of the equation $x^2+2ax-b^2 = 0$ and their ordinates are the roots of the equation $y^2+2py-q^2 = 0$ then find the equation of the circle for which $\overline{AB}$ is a diameter. **[March 2014]**
2. Find the equation of the tangent of $x^2+y^2-2x+4y = 0$ at (3,-1). Also find the equation of the other parallel to it. **[May 2017TS]**
3. Show that the line $x+y+1 = 0$ touches the circle $x^2+y^2-3x+7y+14 = 0$ and find the point of contact. **[May 2017AP]**
4. Prove that the tangent at (-1,2) to the circle $x^2+y^2-4x-8y+7 = 0$ also touches the circle $x^2+y^2+4x+6y = 0$ and find the point of contact. **[June 2010]**
5. If a point P is moving such that the lengths of tangent drawn from P to $x^2+y^2-4x-6y-12 = 0$ and $x^2+y^2+6x+18y+26 = 0$ are in the ratio 2:3 then find the equation of the locus of P. **[May 2013 March 2019AP 2017TS]**
6. Find the length of the chord
   i). $x-y-3 = 0$ of the circle $x^2+y^2-x+3y-22 = 0$ **[March 20TS 18AP May 16AP]**
   ii). $x+y+1 = 0$ of the circle $x^2+y^2-8x-2y-8 = 0$ **[March 2016TS]**
   iii). $x\cos\alpha + y\sin\alpha = p$ of the circle $x^2+y^2 = a^2$ **[May 2016TS]**
7. i).Find the equation of the circle with centre (-2,3) and which cuts off a chord of the length '2' on the line $3x+4y+4 = 0$ **[May 2018AP March 2011]**
   ii).If the line $y = mx+c$ and the circle $x^2+y^2 = a^2$ intersects at A and B and $AB = 2\lambda$ then show that $c^2 = (1+m^2)(a^2-\lambda^2)$ **[March 2019TS]**
8. Find the area of the triangle formed by the normal at (3,-4) to the circle $x^2+y^2-22x-4y+25 = 0$ with the coordinate axes **[March 2018TS]**
9. Find the pole of the line
   i).$x+y+2 = 0$ with respect to the circle $x^2+y^2-4x+6y-12 = 0$ **[March 2017AP May 2015AP]**
   ii).$3x+4y-45 = 0$ with respect to the circle $x^2+y^2-6x-8y+5 = 0$ **[March 2016AP]**

10. i).Find the value of k if the lines $x+y-5=0$, $2x+ky-8=0$ are conjugate with respect to the circle $x^2+y^2-2x-2y-1=0$ **[May 2018TS]**
ii).Find the value of k if the lines $kx+3y-1=0$, $2x+y+5=0$ are conjugate with respect to the circle $x^2+y^2-2x-4y-4=0$ **[May 2015TS]**

11. Find the coordinates of the point of intersection of tangents at the points where $x+4y-14=0$ meets the circle $x^2+y^2-2x+3y-5=0$ **[March 2016AP]**

12. i).Find the pair of tangents drawn from (1,3) to the circle $x^2+y^2-2x+4y-11=0$ and find the anlge between them. **[March 2016TS]**
ii).Find the angle between the pair of tangents drawn from (3,2) to the circle $x^2+y^2-6x+4y-2=0$ **[March 2012]**

13. Find the midpoint of the chord $x-2y+7=0$ with respect to the circle $x^2+y^2-2x-10y+1=0$. Also find the length of the chord. **[May 2014]**

14. Find the pair of tangents from the origin to the circle $x^2+y^2+2gx+2fy+c=0$ and hence deduce a condition for these tangents to be perpendicular. **[May 2016TS 2012]**

15. Find the equation of the tangent at the poit $30^0$ (parametric value of $\theta$) of the circle $x^2+y^2+4x+6y-39=0$ **[May 2019TS]**

16. If the two circles $x^2+y^2+2gx+2fy=0$, $x^2+y^2+2g^1x+2f^1y=0$ touch each other, prove that $fg^1=f^1g$ **[March 2020AP]**

## QNO : 12 IN IPE

## 2. SYSTEM OF CIRCLES

1. Find the equation of the circle which passes through the origin and cutting orthogonally the two circles $x^2+y^2-4x+6y+10=0$, $x^2+y^2+12y+6=0$ **[May 2018AP]**

2. Find the equation of the circle which cuts orthogonally the three circles
i). $x^2+y^2+2x+17y+4=0$, $x^2+y^2+7x+6y+11=0$, $x^2+y^2-x+22y+3=0$ **[May 2008]**
ii). $x^2+y^2+2x+4y+1=0$, $2x^2+2y^2+6x+8y-3=0$, $x^2+y^2-2x+6y-3=0$ **[May 2012]**

3. i). Find the equation of the circle which passes through the point (0, -3) and the intersects the circles $x^2+y^2-6x+3y+5=0$, and $x^2+y^2-x-7y=0$ orthogonally. **[May 2015TS 2013]**
ii). Find the equation of the circle which cuts the circles $x^2+y^2-4x-6y+11=0$, and $x^2+y^2-10x-4y+21=0$ orthogonally and has the diameter along the straight line $2x+3y=7$ **[March 2016AP]**

4. Find the equation of the circle cutting orthogonally the circle $x^2+y^2-4x+2y+4=0$, and passing through the origin and which has its centre on the line $x+y=4$. **[April 2001]**

5. Find the equation of the circle which passes through the points (2,0), (0,2) and orthogonal to the circle $2x^2+2y^2+5x-6y+4=0$ **[May 2019TS]**

6. Find the radical centre of the circles and also find the equation of the circle which cuts orthogonally the three circles $x^2+y^2+3x+2y+1=0$, $x^2+y^2-x+6y+5=0$, $x^2+y^2+5x-8y+15=0$ **[March 2007]**

7. Find the equation of the common chord of the circles and also find the length of the common chord.

i). $x^2+y^2+3x+5y+4=0$, $x^2+y^2+5x+3y+4=0$ **[March 2018TS]**

ii). $x^2+y^2+2x+2y+1=0$, $x^2+y^2+4x+3y+2=0$ **[March 2017AP 2017TS May 2015AP]**

8. i). The line $2x+3y=1$ cuts the circle $x^2+y^2=4$ in A and B.Find the equation of the circle on AB as diameter. **[May 2014]**

ii). If the line $x+y=3$ is the equation of chord $\overline{AB}$ of the circle $x^2+y^2-2x+4y-8=0$, find the equation of the circle having $\overline{AB}$ as a diameter. **[May 2017AP 2016AP March 2015AP]**

9. Find the equation to the circle passing through the points of intersection of the the circles $x^2+y^2-8x-6y+21=0$, $x^2+y^2-2x-15=0$ and (1,2) **[May 2017TS March 2019AP]**

## QNO : 13 & 14 IN IPE

## 4. ELLIPSE

1. If P is a point on the ellipse $x^2/a^2+y^2/b^2=1$ with foi S and $S^1$ then $PS+PS^1=2a$ **[March 2013]**

2. Find the equation of the ellipse whose focus is (1,-1), eccentricity is 2/3 and directrix is $x+y+2=0$. **[May 2016AP March 2019TS]**

3. Find the equation of the ellipse whose axes are the coordinates axes respectively and

i). length of latus rectum = 15/2, distance between the foci = 2 **[March2018AP 2015TS]**

ii). length of latus rectum = 4, distance between foci = $4\sqrt{2}$ **[March 2019AP 2018TS]**

iii). Distance between the foci is 8, and distance between the directrices is 32 **[May 2017AP]**

iv). passes through the points (-2,2) and (3,-1) **[March 2017TS]**

4. Find the centre, eccentricity, vertices, foci, length of latus rectum and the equations of directrices of the ellipse.

i). $9x^2+16y^2=144$ **[May 2019TS 2017TS March 2020TS 2017AP 2016TS 2014]**

ii). $x^2+2y^2-4x+12y+14=0$ **[May 2007]**

iii). $9x^2+16y^2-36x+32y-92=0$ **[May 2018TS 2015TS March 2018TS 2015TS]**

iv). $4x^2+y^2-8x+2y+1=0$ **[May 2018AP March 2011 2010]**

v). $3x^2+y^2-6x-2y-5=0$ **[May 2015AP]**

5. S and T are the foci of an ellipse and B is one end of the minor axes. If STB is an equilateral triangle, then find the eccentricity of the ellipse. **[March 2020AP]**

6. Find the equations of the tangent and normal to the ellipse $x^2+8y^2=33$ at (-1,2) **[May 16TS March 2020TS]**

7. Find the equations of the tangents to the ellipse $2x^2+y^2=8$ which

i). are parallel to $x-2y-4=0$

ii). are perpendicular to $x+y+2=0$ **[May 2019TS 2017AP March 2017TS]**

8. Find the equations of the tangent and normal to the ellipse

i). $2x^2+3y^2=11$ at the point whose ordinate is 1. **[March 2019TS 2016TS]**

ii). $9x^2+16y^2=144$ at the end of latusrecta in the first quadrant. **[March 2015AP]**

9. Find the condition that the line

i). $lx+my+n=0$ may be a tangent to the ellipse $x^2/a^2+y^2/b^2=1$. **[May 2015AP]**

ii).x $\cos\alpha$ +y $\sin\alpha$ = p may be a tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$ **[March 2020AP 2014]**

iii).Find the value of k if 4x+y+k = 0 is a tangent to the ellipse $x^2 + 3y^2 = 3$

**[March 2016AP 2015AP]**

10. Show that the points of intersection of the perpendicular tangents to an ellipse lies on a circle. **[March 2016AP May 2011]**

11. Show that the locus of the feet of the perpendiculars drawn from foci to any tangent of the ellipse is the auxiliary circle. **[March 2019AP 2017AP]**

12. If the normal at the end of latus rectum of an ellipse $x^2/a^2 + y^2/b^2 = 1$ passes through one end of the minior axis then show that $e^4+e^2 = 1$ **[May 2017TS 2014]**

13. Find the equations of the tangents to $9x^2 + 16y^2 = 144$, which make equal intercepts on the coordinate axes. **[May 2015TS]**

14. A circle of radius 4, is concentric with the ellipse $3x^2+13y^2 = 78$. Prove that a common tangent is inclined to the major axis at an angle $45^0$. **[May 2018AP]**

15. If a tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$ (a>b) meets its major and minor axis at M and N respectively then prove that $\frac{a^2}{(CM)^2} + \frac{b^2}{(CN)^2} = 1$ where C is the centre of the ellipse.

**[May 2018TS March 2018AP]**

16. The tangent and normal to the ellipse $x^2+4y^2 = 4$ at a point P($\theta$) on it meets the major axis in Q and R respectively. If $0<\theta<\pi/2$ and QR = 2 then show that $\theta = \text{Cos}^{-1}(2/3)$.

**[May 2016AP 2014]**

## QNO : 15 IN IPE

## 5. HYPERBOLA

1. Find the eccentricity, foci, directrices, length of the latus rectum of the hyperbola

i). $x^2-4y^2 = 4$. **[May 2016AP March 2020AP 2019TS 2018AP 2016AP]**

ii). $16y^2-9x^2 = 144$ **[March 2018AP May 2017AP]**

iii). $5x^2-4y^2+20x+8y = 4$ **[May 2012]**

2. Find the equation to the hyperbola whose foci are (4,2) and (8,2) and eccentricity 2. **[March 2009]**

3. The condition that the line y = mx+c may be a tangent to the hyperboal $x^2/a^2 - y^2/b^2 = 1$ is $c^2 = a^2m^2-b^2$. **[March 2003]**

4. If the line $lx+my+n = 0$ is a tangent to the hyperboal $x^2/a^2 - y^2/b^2 = 1$ then show that $a^2l^2 - b^2m^2 = n^2$. **[March 2007]**

5. If the line $lx+my = 1$ is a normal to the hyperboal $x^2/a^2 - y^2/b^2 = 1$ then show that $a^2/l^2 - b^2/m^2 = (a^2+b^2)^2$

6. Find the equations of the tangents to the hyperbola $3x^2-4y^2 = 12$ which are

i). parallel

ii). perpendicular to the line y = x - 7.

**[May 2018TS 2015AP 2015TS March 2020TS 2017AP 2015AP]**

7. Find the equations of the tangents to the hyperbola $x^2-4y^2=4$ which are
   i). parallel
   ii). perpendicular to the line $x+2y=0$
   **[March 2019AP 2015TS 2014 2011 May 2014 2013]**

8. The angle between the asymptotes of the hyperboal $x^2/a^2 - y^2/b^2 = 1$ is $2\,\text{Tan}^{-1}\frac{b}{a}$ or $2\,\text{Sec}^{-1}e$
   **[May 2012 March 2018TS]**

9. The point of intersection of two perpendicular tangents to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ lies on the circle $x^2+y^2 = a^2-b^2$ **[May 2017TS March 2016TS]**

10. Tangents to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ makes angles $\theta_1$ and $\theta_2$ with transverse axis of a hyperbola.Show that the point of intersection of their tangents lies on the curve $2xy = k(x^2-a^2)$ when $\tan\theta_1+\tan\theta_2 = k$ **[May 2019TS 2018AP 2016TS]**

# QNO : 16 IN IPE
# 7. DEFINITE INTEGRALS

Evaluate

1. $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}\,dx$ **[May 2015TS March 2017AP]**

2. i). $\int_0^{\pi/2} \frac{1}{4+5\cos x}\,dx$ **[May 2018TS March 2016AP 2015AP]**

   ii). $\int_0^{\pi} \frac{1}{3+2\cos x}\,dx$ **[March 2015AP]**

3. $\int_a^b \sqrt{(x-a)(b-x)}\,dx$ **[March 2018TS]**

4. i). $\int_0^{\pi/2} \frac{\cos^{5/2} x}{\sin^{5/2} x+\cos^{5/2} x}\,dx$ **[May 2015AP]**

   ii). $\int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x+\cos^5 x}\,dx$ **[March 17AP 14]**

   iii). $\int_0^{\pi/2} \frac{\sin^2 x-\cos^2 x}{\sin^3 x+\cos^3 x}\,dx$ **[May 2012]**

   iv). $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ **[March 20TS 14]**

   v). $\int_0^{\pi/2} \frac{a\sin x+b\cos x}{\sin x+\cos x}\,dx$ **[May 2017AP]**

5. If $I_n = \int_0^{\pi/2} \sin^n x\,dx$ then $I_n = \frac{n-1}{n} I_{n-2}$ **[March 2015TS]**

6. If $I_n = \int_0^{\pi/2} \cos^n x\,dx$ then $I_n = \frac{n-1}{n} I_{n-2}$ **[April 2000]**

7. If $I_n = \int_0^{\pi/4} \tan^n x\,dx$ then $I_n = \frac{1}{n-1} - I_{n-2}$ **[March 2006]**

8. Find the area between the parabola $y = x^2$ and the line $y = 2x$ **[May 2013]**

9. i).Find the area enclosed between the curve $y = x^2-5x$, $y = 4-2x$ **[March 2013]**
   ii).Find the area enclosed between the curve $y = 4x-x^2$, $y = 5-2x$ **[March 2016TS]**
   iii).Find the area enclosed between the curve $y = x^2+1$, $y = 2x-2$, $x = -1$, $x = 2$ **[May 2016AP]**

10. i).Find the area enclosed between the curve $y = x^2$, $y = x^3$ **[May 2019TS]**
    ii).Find the area enclosed between the curves $y = 2-x^2$, and $y = x^2$ **[April 2001]**
    iii).Find the area enclosed between the curves $y = \sqrt{x}$, $y = x^2$ **[March 2018TS]**
    iv).Find the area of the region bounded by the parabolas $y^2 = 4x$, and $x^2 = 4y$ **[May 2014 March 2020AP 2017TS]**
    v).Find the area bounded by the parabolas $y^2 = 4ax$, and $x^2 = 4by$ **[March 2003]**
    vi).Find the area enclosed between $y^2 = 4x$, and $y^2 = 4(4-x)$ **[May 2019TS March 2011]**

11. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$. Also deduce the area of the circle $x^2+y^2 = a^2$ **[May 2017AP March 2014]**

12. Find the area bounded by the curves $y = \sin x$ and $y = \cos x$ between any two consecutive points of intersections. **[March 2018AP]**

13. Evaluate

    i) $\lim_{n\to\infty} \frac{1+2^4+3^4+.....+n^4}{n^5}$ **[March 2020AP]**

    ii). $\lim_{n\to\infty} \frac{2^k+4^k+6^k+.....+(2n)^k}{n^{k+1}}$ **[May 2018AP]**

# QNO : 17 IN IPE
# 8. DIFFERENTIAL EQUATIONS

Solve

1. i). $(xy^2+x)dx+(yx^2+y)dy = 0$ **[May 2013 March 2020TS 2015AP]**
   ii). $(xy+x)\,dy = (xy+y)\,dx$ **[May 2016TS]**

2. $(e^x+1)y\,dy+(y+1)\,dx = 0$ **[June 2010]**

3. $\frac{dy}{dx} = \left|\frac{x(2\log x+1)}{\sin y+y\cos y}\right|$ **[March 2005]**

4. $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1} = 0$ **[May 2008]**

5. i). $\text{Sin}^{-1}\frac{dy}{dx} = x+y$ **[March 2020TS]**

   ii). $\frac{dy}{dx}+1 = e^{x+y}$ **[March 2018TS]**

   iii). $\frac{dy}{dx} - x\tan(y-x) = 1$ **[May 2015TS]**

6. i). $\frac{dy}{dx}+\frac{4x}{1+x^2}y=\frac{1}{(1+x^2)^2}$ [May 2017AP]

ii). $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$ [March 2006]

7. $\frac{1}{x}\frac{dy}{dx}+ye^x=e^{(1-x)e^x}$ [May 2018AP]

8. $x\log x\frac{dy}{dx}+y=2\log x$ [May 2014 March 2020AP]

9. i). $\frac{dy}{dx}+y\sec x=\tan x$ [June 2010]

ii). $\frac{dy}{dx}+y\tan x=\cos^3 x$ [May 2018TS 2011 March 2017AP]

iii). $\frac{dy}{dx}-y\tan x=e^x\sec x$ [May 2019TS]

iv). $(1+x^2)\frac{dy}{dx}+y=e^{\text{Tan}^{-1}x}$ [March 2018AP 2016AP 2015TS 2010 May 2013]

v). $(1+x^2)\frac{dy}{dx}+y=\text{Tan}^{-1}x$ [May 2016AP]

vi). $\frac{dy}{dx}+y\tan x=\sin x$ [March 2016TS 2012]

vii). $\cos x\frac{dy}{dx}+y\sin x=\sec^2 x$ [March 2019TS 2014]

viii). $x(x-1)\frac{dy}{dx}-y=x^3(x-1)^3$ [March 2019AP]

10. $x(x-2)\frac{dy}{dx}-2(x-1)y=x^3(x-2)$ given that $y=9$ when $x=3$. [May 2017TS]

11. i). $(x+y+1)\frac{dy}{dx}=1$ [March 2017TS 2013]

ii). $(x+2y^3)\frac{dy}{dx}=y$ [May 2012]

iii). $(1+y^2)\,dx=(\text{Tan}^{-1}y-x)\,dy$ [March 2018TS 2015AP May 2015AP 2015TS]

# QNO : 18 IN IPE

# 1. CIRCLES

1. Find the equation of the circle passing through

i). (1,1), (2,-1), (3,2) [May 2016TS]

ii). (3,4), (3,2), (1,4) [March 2018AP May 2016AP 2016TS 2013]

iii). (1,2), (3,-4), (5,-6) [May 2018TS March 2016TS]

iv). (2,1), (5,5), (-6,7) [May 2018AP]

2. Find the equation of the circle passing through the points
   i).(4,1), (6,5) and having the centre on the line $4x+y-16=0$ **[April 99, May 97]**
   ii).(2,-3), (-4,5) and having the centre on the line $4x+3y+1=0$
   **[May 19TS 17TS March 20TS]**
   iii).(-2,3), (4,5) and whose centre lies on x-axis. **[March 2015AP 2015TS 2010]**
   iv).(4,1), (6,5) and whose centre lies on $4x+3y-24=0$
   **[March 2020AP 2018TS 2016AP 2014 2012]**
3. Show that the following points are concyclic and find the equation of the circle on which they lie
   i).(1,1), (-6,0), (-2,2), (-2,-8) **[May 2017AP March 2019AP]**
   ii).(1,2), (3,-4), (5,-6), (19,8) **[May 2015TS]**
   iii).(9,1), (7,9), (-2,12), (6,10) **[May 2008 March 2019TS]**
4. Find the value of 'c' so that (2,0), (0,1), (4,5), (0,c) are concyclic.
   **[May 2015AP 2014 March 2017AP 2017TS 2015AP 2015TS]**

# QNO : 19 IN IPE
# 1. CIRCLES

1. Show that the following circles touch each other. Find the point of contact and the equation of the tangent at the point of contact.
   i). $x^2+y^2-4x-6y-12=0$, $x^2+y^2+6x+18y+26=0$ **[March 2017AP 2013]**
   ii). $x^2+y^2-6x-9y+13=0$, $x^2+y^2-2x-16y=0$ **[May 18TS 18AP March 20TS 18TS]**
   iii). $x^2+y^2-8x-2y+8=0$, $x^2+y^2-2x+6y+6=0$ **[March 2014]**
   iv). $x^2+y^2-4x-6y-12=0$, $5(x^2+y^2)-8x-14y-32=0$ **[May 2017TS]**
   v).$x^2+y^2-6x-2y+1=0$, $x^2+y^2+2x-8y+13=0$ **[March 2016AP 2011 2010 May 2017AP 2016AP]**
   vi). $x^2+y^2+6x+2y-90=0$, $x^2+y^2-2x-4y-20=0$ **[March 2015TS]**
2. Find the equation of the circle whose radius is 5 and which touches the circle $x^2+y^2-2x-4y-20=0$ at the point (5,5) **[May 2016TS March 2020AP]**
3. Show that four common tangents can be drawn for the circles given by $x^2+y^2-14x+6y+33=0$, $x^2+y^2+30x-2y+1=0$ and find the internal and external centres of similitude. **[March 2019TS]**
4. Find the equations to the direct common tangents to the circles $x^2+y^2+22x-4y-100=0$, $x^2+y^2-22x+4y+100=0$ **[May 2015TS March 2018AP 2015TS]**
5. Find the transverse common tangents of the circles $x^2+y^2-4x-10y+28=0$, and $x^2+y^2+4x-6y+4=0$ **[May 2019TS March 2019AP 2017TS 2015AP 2014]**
6. Find the equations of common tangents to the circles $x^2+y^2-2x-6y+6=0$, $x^2+y^2=1$
   **[May 2015AP]**

# QNO : 20 IN IPE
# 3. PARABOLA

1. Derive Standard from of the Parabola
   **[May 2018TS 2015AP March 2020TS 2018TS 2017AP 2017TS 2016TS 2015AP]**

2. Find the vertex, focus, latus rectum and the equations of axis and directrix of the parabola $y^2-x+4y+5=0$ **[March 2005]**

3. Find the equation of the parabola whose axis is parallel to x-axis and passing through (-2,1), (1,2), (-1,3). **[May 2018AP 2017TS 2016AP]**

4. Find the equation of the parabola whose axis is parallel to y-axis and passing through (4,5), (-2,11), (-4,21) **[May 2012]**

5. Find the equation of the parabola whose focus is (3,5) and vertex is (1,3) **[March 2019AP]**

6. Show that the equations of common tangents of $x^2+y^2=2a^2$ and $y^2=8ax$ are $y=\pm(x+2a)$ **[May 2019TS 2017AP 2016TS March 2010]**

7. Show that the common tangent to the parabola $y^2=4ax$ and $x^2=4by$ is $xa^{1/3}+yb^{1/3}+a^{2/3}b^{2/3}=0$ **[March 2016AP]**

8. Show that the condition that the line $y=mx+c$ to be a tangent to the parabola $x^2=4ay$ is $c=-am^2$ **[March 2012]**

9. Prove that the two parabolas $y^2=4ax$ and $x^2=4by$ intersect at an angle of $\text{Tan}^{-1}\left(\frac{3a^{1/3}b^{1/3}}{2(a^{2/3}+b^{2/3})}\right)$ **[March 2014]**

10. i).From an external point P, tangents are drawn to the parabola $y^2=4ax$ and these tangents make angles $\theta_1,\theta_2$ with its axis such that $\tan\theta_1+\tan\theta_2$ is a constant 'b'.Then show that P lies on the line $y=bx$. **[March 2020AP]**

ii).From an external point P, tangents are drawn to the parabola $y^2=4ax$ and these tangents make angles $\theta_1,\theta_2$ with its axis such that $\cot\theta_1+\cot\theta_2$ is a constant d.Then show that all such P lie on a horizontal line. **[March 2019TS]**

11. Prove that the area of the triangle inscribed in the parabola $y^2=4ax$ is $\frac{1}{8a}|(y_1-y_2)(y_2-y_3)(y_3-y_1)|$ where $y_1, y_2$, and $y_3$ are the ordinates of its vertices **[May 2015TS]**

12. Prove that the area of the triangle formed by the tangents at $(x_1,y_1)$, $(x_2,y_2)$ and $(x_3,y_3)$ to the parabola $y^2=4ax$ is $\frac{1}{16a}|(y_1-y_2)(y_2-y_3)(y_3-y_1)|$ sq units. **[March 2018AP 2015TS]**

## QNO : 21 IN IPE
## 6. INDEFINITE INTEGRATION

Evaluate

1. $\int\frac{x+1}{x^2+3x+12}dx$ **[March 2017AP 2012 May 2016AP]**

2. i). $\int\frac{2x+5}{\sqrt{x^2-2x+10}}dx$ **[May 2017AP March 2015TS]**

ii). $\int\sqrt{\frac{5-x}{x-2}}dx$ **[May 2017TS March 2004]**

3. i). $\int (3x-2)\sqrt{2x^2-x+1}\,dx$ **[May 2015TS]**

ii). $\int (6x+5)\sqrt{6-2x^2+x}\,dx$ **[May 2018TS 2018AP March 2009]**

4. i). $\int \frac{1}{(1+x)\sqrt{3+2x-x^2}}\,dx$ **[May 2014 March 2020TS]**

ii). $\int \frac{1}{(1-x)\sqrt{3-2x-x^2}}\,dx$ **[July 2001]**

iii). $\int \frac{1}{(x+1)\sqrt{2x^2+3x+1}}\,dx$ **[March 2018TS]**

5. i). $\int \frac{dx}{4+5\sin x}$ **[March 2005]** ii). $\int \frac{dx}{5+4\cos x}$ **[March 2012]**

iii). $\int \frac{dx}{4\cos x+3\sin x}$ **[March 2018TS]** iv). $\int \frac{dx}{1+\cos x+\sin x}$ **[March 2020AP 2015TS]**

v). $\int \frac{dx}{3\cos x+4\sin x+6}$ **[May 2015AP]** vi). $\int \frac{dx}{5+4\cos 2x}$ **[May 2013 March 2011]**

vii). $\int \frac{dx}{2-3\cos 2x}$ **[June 2010]**

6. i). $\int \frac{2\cos x+3\sin x}{4\cos x+5\sin x}dx$ **[May 2019TS 2016TS March 2018AP 2015AP]**

ii). $\int \frac{9\cos x-\sin x}{4\sin x+5\cos x}dx$ **[March 2017TS 2008]**

7. i). $\int \frac{\cos x+3\sin x+7}{\cos x+\sin x+1}dx$ **[May 2006 March 2019AP]**

ii). $\int \frac{2\sin x+3\cos x+4}{3\sin x+4\cos x+5}dx$ **[March 2016AP 2016TS 2014 2011]**

# QNO : 22 IN IPE
# 6. INDEFINITE INTEGRATION

1. If $I_n = \int \sin^n x\,dx$ then $I_n = \frac{-\sin^{n-1}x\cos x}{n} + \frac{n-1}{n} I_{n-2}$ where n is a positive integer. and hence deduce that $\int \sin^4 x\,dx$. **[March 2020TS 2017AP 2014 2013 May 2018TS 2015AP]**

2. If $I_n = \int \cos^n x\,dx$ then $I_n = \frac{\cos^{n-1}x\sin x}{n} + \frac{n-1}{n} I_{n-2}$ where n is a positive integer. and hence deduce that $\int \cos^5 x\,dx$. **[May 2019TS 2018AP 2016TS March 2020AP]**

3. If $I_n = \int \tan^n x\,dx$ then $I_n = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$ where n is a positive integer. and hence deduce that $\int \tan^6 x\,dx$. **[May 2017TS 2016AP 2013 March 2018AP 2015AP 2012]**

4. If $I_n = \int \cot^n x\,dx$ then $I_n = -\frac{\cot^{n-1} x}{n-1} - I_{n-2}$ where n is a positive integer. and hence deduce that $\int \cot^4 x\,dx$. **[March 2019TS 2017TS 2016AP]**

5. If $I_n = \int \sec^n x\,dx$ then $I_n = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} I_{n-2}$ where n is a positive integer. and hence deduce that $\int \sec^5 x\,dx$. **[May 2017AP 2015TS]**

6. If $I_n = \int \csc^n x\,dx$ then $I_n = \frac{-\csc^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} I_{n-2}$ where n is a positive integer. and hence dedue that $\int \csc^5 x\,dx$. **[March 2019AP 2016TS May 2014]**

# QNO : 23 IN IPE

# 7. DEFINITE INTEGRALS

1. Evaluate

i). $\int_0^{\pi/4} \log(1+\tan x)\,dx$ **[May 2018AP 2016TS March 2019AP 2016AP]**

ii). $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$ **[May 2019TS 2017TS March 2020TS]**

2. i). $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}\,dx$ **[May 2015TS]**

ii). $\int_0^{\pi/2} \frac{x\,dx}{\sin x+\cos x}$ **[March 2020AP 2018AP 2017TS 2012]**

iii). $\int_0^{\pi} \frac{x}{1+\sin x}\,dx$ **[May 2011]**

iv). $\int_0^{\pi} \frac{x\sin x}{1+\sin x}\,dx$ **[March 2016TS 2015AP 2013 May 2018AP 2015AP]**

v). $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$ **[May 18TS 16AP 14]**

vi). $\int_0^{\pi} \frac{x\sin^3 x}{1+\cos^2 x}\,dx$ **[March 2015TS 2011]**

3. $\int_0^{\pi} x\sin^3 x\,dx$ **[April 2001]**

4. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}\,dx$ **[March 2004]**

5. $\int_0^{\pi} x\sin^7 x\cos^6 x\,dx$ **[March 2019TS]**

6. $\int_{-a}^{a} x^2(a^2-x^2)^{3/2}\,dx$ **[May 2017TS]**

# QNO : 24 IN IPE
# 8. DIFFERENTIAL EQUATIONS

Solve the following differential equations

1. i). $(x+y)\,dy = (x-y)\,dx$ **[May 2008]**
   ii). $(2x-y)\,dy = (2y-x)\,dx$ **[March 2012]**
2. i). $(x^2+y^2)\,dx = 2xy\,dy$ **[March 2020AP 2017AP 2016AP]**
   ii). $(x^2-y^2)\,dx - xy\,dy = 0$ **[May 2017AP 2009]**
   iii). $(x^2-y^2)\frac{dy}{dx} = xy$ **[May 2011]**
   iv). $(x^2-xy)\,dy = (y^2-2xy)\,dx$ **[March 2019AP]**
   v). $(x^2-2xy)\,dy = (y^2-2xy)\,dx$ **[July 2001]**
   vi). $2x^2\,dy = (x+y)^2\,dx$ **[March 2005]**
3. i). $xy^2\,dy - (x^3+y^3)\,dx = 0$ **[May 2018TS]**
   ii). $(x^2y-2xy^2)\,dx = (x^3-3x^2y)\,dy$ **[March 2018AP]**
   iii). $(x^3-3xy^2)\,dx + (3x^2y-y^3)\,dy = 0$ **[May 2018AP 2014]**
4. i). $x\,dy = [y + x\cos^2(y/x)]\,dx$ **[March 2013 2011]**
   ii). Find the equation of a curve whose gradient is $\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x}$, where $x>0$, $y>0$ and which passes through the point $(1, 45^0)$ **[May 2016AP 2016TS]**
   iii). Give the solution of $x\sin^2(y/x)\,dx = y\,dx - x\,dy$ and which passes through the point $(1, 45^0)$ **[March 2014]**

Solve the differential equations

5. i). $(2x-2y+5)\,dy = (x-y+3)\,dx$ **[May 2015AP]**
   ii). $(2x+y+1)\,dx + (4x+2y-1)\,dy = 0$ **[March 2015TS]**
   iii). $(2x+4y+3)\,dy = (2y+x+1)\,dx$ **[March 2019TS]**
   iv). $(3y+2x+4)\,dy = (4x+6y+5)\,dx$ **[May 2019TS]**
6. $\frac{dy}{dx} = \frac{3y-7x+7}{3x-7y-3}$ **[March 2016TS]**
7. Solve $\frac{dy}{dx}(x^2y^3+xy) = 1$ **[March 2011]**



ALL THE BEST