



Total No. of Questions – 24

Regd.

Total No. of Printed Pages – 4

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Part – III

MATHEMATICS, Paper – I (A)

(English Version)

Time : 3 Hours]

[Max. Marks : 75

Note : This question paper consists of three Sections – A, B and C.

SECTION – A

10 × 2 = 20

I. Very Short Answer Type questions :

- (i) Answer all the questions.
(ii) Each question carries two marks.

1. If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$ then find $(g \circ f)(x)$.

2. If $f = \{(1, 2), (2, -3), (3, -1)\}$ then find (i) $2f$ (ii) f^2 .

3. Find the trace of the matrix $A = \begin{bmatrix} 1 & 3 & -5 \\ 2 & -1 & 5 \\ 2 & 0 & 1 \end{bmatrix}$.

4. Find the rank of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$.

5. If $\vec{a} = 2\vec{i} + 5\vec{j} + \vec{k}$ and $\vec{b} = 4\vec{i} + m\vec{j} + n\vec{k}$ are collinear vectors then find m, n .

6. Find the vector equation of the plane passing through the points $(0, 0, 0)$, $(0, 5, 0)$ and $(2, 0, 1)$.
7. Find the angle between the planes $\vec{r} \cdot (2\vec{i} - \vec{j} + 2\vec{k}) = 3$ and $\vec{r} \cdot (3\vec{i} + 6\vec{j} + \vec{k}) = 4$.
8. If $\tan 20^\circ = \lambda$ then show that $\frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \tan 110^\circ} = \frac{1 - \lambda^2}{2\lambda}$.
9. Find the range of $7 \cos x - 24 \sin x + 5$.
10. Prove that $(\cosh x - \sinh x)^n = \cosh(nx) - \sinh(nx)$.

SECTION - B

5 × 4 = 20

II. Short Answer Type questions :

- (i) Answer any **five** questions.
- (ii) Each question carries **four** marks.

11. If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ then show that $AA' = A'A = I$.
12. If the points whose position vectors are $3\vec{i} - 2\vec{j} - \vec{k}$, $2\vec{i} + 3\vec{j} - 4\vec{k}$, $-\vec{i} + \vec{j} + 2\vec{k}$ and $4\vec{i} + 5\vec{j} + \lambda\vec{k}$ are coplanar then show that $\lambda = \frac{-146}{17}$.
13. Find the vector area and area of the parallelogram having $\vec{a} = \vec{i} + 2\vec{j} - \vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} + 2\vec{k}$ as adjacent sides.
14. Prove that $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} = \frac{3}{2}$.
15. Solve $7 \sin^2 \theta + 3 \cos^2 \theta = 4$.

16. Prove that $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$.

17. If $\cot \frac{A}{2}, \cot \frac{B}{2}, \cot \frac{C}{2}$ are in A.P. then prove that a, b, c are in A.P.

SECTION - C

$5 \times 7 = 35$

III. Long Answer Type questions :

(i) Answer any **five** questions.

(ii) Each question carries **seven** marks.

18. (a) If $f(x) = \frac{x+1}{x-1}$, ($x \neq \pm 1$) then find $(f \circ f \circ f)(x)$.

(b) If $f : A \rightarrow B$, $g : B \rightarrow C$, $h : C \rightarrow D$ are functions then show that $h \circ (g \circ f) = (h \circ g) \circ f$.

19. Show that $49^n + 16n - 1$ is divisible by 64 for all positive integers by using Mathematical induction.

20. Show that $\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3$.

21. Solve the equations using Cramer's Rule $x + y + z = 1$, $2x + 2y + 3z = 6$, $x + 4y + 9z = 3$.

22. Find the shortest distance between the skew lines

$$\vec{r} = (6\vec{i} + 2\vec{j} + 2\vec{k}) + t(\vec{i} - 2\vec{j} + 2\vec{k}),$$

$$\text{and } \vec{r} = (-4\vec{i} - \vec{k}) + s(3\vec{i} - 2\vec{j} - 2\vec{k}).$$

23. If $A + B + C = \frac{\pi}{2}$ then prove that $\cos 2A + \cos 2B + \cos 2C = 1 + 4 \sin A \sin B \sin C$.

24. Prove that $r + r_3 + r_1 - r_2 = 4R \cos B$.